A Systematic Formulation for Dynamics of Flexible Multibody Systems with Topology Changes

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EXTENDED ABSTRACT

1 Introduction

Mechanical systems with time-varied topology are very common in engineering. With changes of structure such as locks and releases of joints, those systems are subjected to the impulses of the constraint forces, causing the change of their motion. Topology changes usually occur in a very short period. Therefore, it is difficult to be captured in numerical simulations.

There are a variety of methodologies to deal with multibody systems with variable topology [1-7]. Some methods[1-3] model the additional constraints as spring-damping elements. With a proper magnitude of the stiffness and damping coefficient, the relative movement around the locked joints can be well restricted. However, the time step must be set small enough for the equations to converge, which decreases the efficiency of the numerical simulation. Some methods[4-7] treat the topology changes as instantaneous events. The impulse-momentum equations were deduced to efficiently solve discontinuous changes of the generalized velocity. The strategy has been proved useful when concerning rigid multibody systems. However, the influences of the elastic deformation on the topology change have not been fully discussed.

In this paper, a systematic formulation is presented to simulate the topology changes of flexible multibody systems. Based on the mode coordinates obtained from the finite element analysis and the impulse-momentum equations, this method is able to describe the dynamics of systems with variable topology, such as constraint addition and deletion.

2 Methodology

In order to describe the configuration of a flexible body in space, the body-attached reference frame and elastic coordinates are required[7]. Let $\boldsymbol{r^{ki}}$ be the position vector of an arbitrary point k on the ith flexible body and it can be written as

$$\boldsymbol{r}^{ki} = \boldsymbol{r}^{i} + \boldsymbol{A}^{i}\left(\boldsymbol{u}_0^{ki} + \boldsymbol{N}^{ki}\boldsymbol{T}^{i}\boldsymbol{a}^{i}\right) \tag{1}$$

where $\boldsymbol{r^i}$ is the global position of the origin of the ith body reference frame relative to the inertial frame; $\boldsymbol{A^i}$ is the transformation matrix from the inertial frame to the ith body reference frame, and $\boldsymbol{A^i}$ can be expressed by Euler quaternion $\boldsymbol{p}^i = \left[\boldsymbol{p}_0^i, \boldsymbol{p}_1^i, \boldsymbol{p}_2^i, \boldsymbol{p}_3^i\right]^{\boldsymbol{T}}$; $\boldsymbol{u}_0^{ki}$ is the undeformed position vector in the body reference frame, $\boldsymbol{N^{ki}}$ is the modified shape function[7]; $\boldsymbol{T^i}$ is the modal transformation matrix[7]; $\boldsymbol{a^i}$ is the vector of the mode coordinates. Therefore, the position of an arbitrary point on the ith flexible body can be represented by a vector $\boldsymbol{q^i}$ including $\boldsymbol{r^i}, \boldsymbol{p^i}$, and $\boldsymbol{a^i}$:

$$\boldsymbol{q}^i = \left[\boldsymbol{r}^{i^T}, \boldsymbol{p}^{i^T}, \boldsymbol{a}^{i^T}\right]^{\boldsymbol{T}} \tag{2}$$

The generalized coordinate $\boldsymbol{q}$ of a system with $\boldsymbol{N_b}$ bodies can be defined as

$$\boldsymbol{q} = \left[\boldsymbol{q}^{1^T}, \boldsymbol{q}^{2^T}, \cdots, \boldsymbol{q}^{N_b{}^T}\right]^{\boldsymbol{T}} \tag{3}$$

By introducing the position level constraint $\boldsymbol{\Phi}(\boldsymbol{q}) = \boldsymbol{0}$ (its Jacobian is denoted as $\boldsymbol{C}$) and the Lagrange multipliers $\boldsymbol{\lambda}$, the dynamic equations of the constrained motions can be written as[4]

$$\boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}} + \boldsymbol{d}(\boldsymbol{q}, \dot{\boldsymbol{q}}) = -\frac{\partial \boldsymbol{V_e}}{\partial \boldsymbol{q}} + \boldsymbol{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{t}) - \boldsymbol{C}^{\boldsymbol{T}}(\boldsymbol{q})\boldsymbol{\lambda} \tag{4}$$

where $\boldsymbol{M(q)}$ is the mass matrix; $\boldsymbol{d}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ stands for the centrifugal, Coriolis and gyroscopic dynamic terms; V_e is the elastic energy generated by the deformation; $\boldsymbol{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{t})$ is the applied external forces.

As shown in Fig. 1, the topological variation starts with an instantaneous change of the constraint equations, resulting in the redistribution of the generalized velocities. We assume that these events happen in such a short time ($t^+ - t^- = \Delta t \to 0$) that during this moment the configuration of the system can be treated as the same.

Taking the integral of Eq. (4) over $t \in [t^-, t^+]$, it can be written as

$$\int_{t_-}^{t_+} \boldsymbol{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}dt + \int_{t_-}^{t_+} \boldsymbol{d}(\boldsymbol{q}, \dot{\boldsymbol{q}})dt = \int_{t_-}^{t_+} \left(-\frac{\partial \boldsymbol{V_e}}{\partial \boldsymbol{q}} + \boldsymbol{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{t})\right)dt - \int_{t_-}^{t_+} \boldsymbol{C}_B^T(\boldsymbol{q})\boldsymbol{\lambda}dt \tag{5}$$

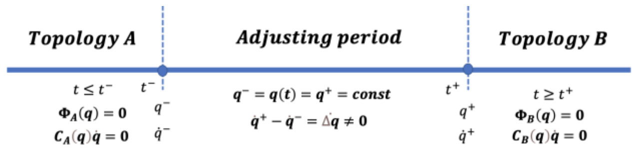

Figure 1: The topology change

Since $\Delta t \to 0$, the impulse of $\boldsymbol{d}(\boldsymbol{q}, \dot{\boldsymbol{q}})$ and $-\frac{\partial V_e}{\partial \boldsymbol{q}} + \boldsymbol{f}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{t})$ are both infinitesimal. Combine with the unity of $\boldsymbol{q}(t)$ at the instant, and let $\boldsymbol{\mu}$ be the impulse of $\boldsymbol{\lambda}$, Eq. (5) can be simplified as

$$\boldsymbol{M}(\boldsymbol{q})\Delta\dot{\boldsymbol{q}} = -\boldsymbol{C}_B^T(\boldsymbol{q})\boldsymbol{\mu} \tag{6}$$

Meanwhile, the jump of the generalized velocity should also conform to the constraints of the topology B, which means

$$\boldsymbol{C}_B(\boldsymbol{q})\dot{\boldsymbol{q}}^+ = \boldsymbol{C}_B(\boldsymbol{q})\left(\Delta\dot{\boldsymbol{q}} + \dot{\boldsymbol{q}}^-\right) = \boldsymbol{0} \tag{7}$$

Associating Eq. (6) with Eq. (7) yields the linear algebraic equations to solve $\Delta\dot{\boldsymbol{q}}$ and $\boldsymbol{\mu}$:

$$\begin{bmatrix} \boldsymbol{M}(\boldsymbol{q}) & \boldsymbol{C}_B^T(\boldsymbol{q}) \\ \boldsymbol{C}_B(\boldsymbol{q}) & \boldsymbol{0} \end{bmatrix} \begin{bmatrix} \Delta\dot{\boldsymbol{q}} \\ \boldsymbol{\mu} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ -\boldsymbol{C}_B(\boldsymbol{q})\dot{\boldsymbol{q}}_- \end{bmatrix} \tag{8}$$

3 Numerical simulation

A test case in [4] is chosen for simulation and comparison. The fourth-order pendulum swing under gravity which will lock up its third revolute joint is presented in detail in [4]. It should be noted that the cross-sectional area given originally is too small to withstand the internal forces if system's flexibility is considered. Therefore, we adjust it from 1$\mathbf{mm^2}$ to 400$\mathbf{mm^2}$. Each body is divided into eight beam elements. The impact of system's flexibility on the topology changes can be reviewed by the differences between the rigid case and the flexible case. The results of the numerical simulation are shown in Fig. 2. It can be seen that the trajectories of the pendulum present an obvious difference between the flexible and rigid systems, and the flexibility of the pendulum can be captured effectively using the proposed method.

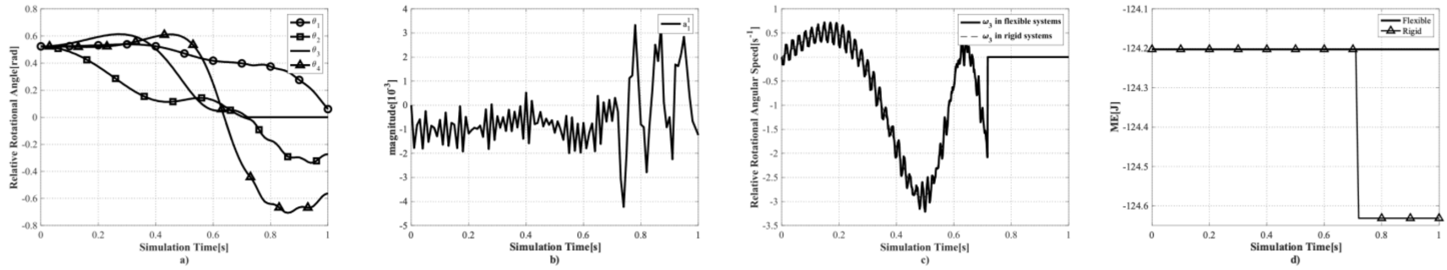

Figure 2: Time history of different variables, which are a) the relative rotational angles b) the first elastic mode coordinate of the first bar c) the angular velocity $\boldsymbol{\omega_3}$ compared with the rigid case d) the mechanical energy compared with the rigid case

4 Conclusion

A new method is described for efficient and accurate simulations of the topology changes in flexible multibody systems. Using this method, the discontinuous change of motion can be calculated using Eq. (8). A numerical test case is given to show the effectiveness of topology changes on motion in flexible multibody systems. Compared with the rigid systems, the results have shown obvious difference from the rigid cases in discontinuous changes of the velocity and the mechanical energy, which results from the system vibration.

5 References

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